Decentralized Multilevel Power Allocation for Random Access

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SUMMARY In this paper, we introduce a distributed power allocation strategy for random access, that has the capabilities of multipacket reception (MPR) and successive interference cancellation (SIC). The proposed random access scheme is suitable for machine-to-machine (M2M) communication application in fifth-generation (5G) cellular networks. A previous study optimized the probability distribution for discrete transmission power levels, with implicit limitations on the successful decoding of at most two packets from a single collision. We formulate the optimization problem for the general case, where a base station can decode multiple packets from a single collision, and this depends only on the signal-to-interference-plus-noise ratio (SINR). We also propose a feasible suboptimal iterative per-level optimization process; we do this by introducing relationships among the different discrete power levels. Compared with the conventional power allocation scheme with MPR and SIC, our method significantly improves the system throughput; this is confirmed by computer simulations.

key words: random access, decentralized power allocation, convex optimization, multipacket reception (MPR), successive interference cancellation (SIC), machine-to-machine (M2M) communications

1. Introduction

Currently, the coming fifth-generation (5G) cellular network is receiving much attention from both academia and industry [1]–[6]. In 5G networks, the number of devices is likely to increase dramatically due to the potential development of Internet of Things (IoT) applications [4]. Consequently, one key requirement of 5G is the native support of machine-to-machine (M2M) communication with a massive number of devices [1]. However, the enormous traffic generated by this M2M communication poses a challenge to the centralized multiple access of current cellular networks; thus, a new random access scheme dealing with such a traffic is required [5].

In conventional random access schemes such as pure and slotted ALOHA, a collision is defined as an event that two or more users transmit simultaneously [7]. Conventional analysis of random access mainly focuses on the performance of the media access control (MAC) layer, ignoring the details of the physical (PHY) layer, and all packets received during the collision time slot are considered to be destroyed [8]. Therefore, when the traffic load becomes relatively high, the throughput of these schemes steeply falls down because of collisions.

However, it has been well-recognized in the literature [9]–[16] that more complex procedures could permit decoding of interfering signals, which is referred to as multipacket reception (MPR). For example, if the power of one of the received packets is sufficiently higher than the other packets in the collision, the strongest one can be correctly decoded while the others are lost, which is known as capture effect [9]. Also, a successive interference cancellation (SIC) decoder might decode more packets by successively subtracting the correctly decoded packets from the collision [17]. The SIC decoder performs as well as a near-optimal decoder while it has much lower linear complexity [18]. The use of SIC has thus been actively investigated in wireless multiple-access scenarios [19]–[23].

Recently, the slotted ALOHA with SIC has been extensively studied [24]–[26]. The contention resolution diversity slotted ALOHA (CRDSA) has been proposed to enhance the throughput [24]. In CRDSA, every packet is transmitted twice with a pointer to the slot where the respective copy is sent. Whenever the receiver can decode the packet without the interference, the pointer is extracted and the potential interference in the corresponding slot is removed by the SIC. Upon iterating this procedure, the CRDSA achieves the low packet loss rate even at moderately high loads. In [25], the number of repetition of each packet was optimized using density evolution. Moreover, in [26], the idea of rateless coding has been applied to design the slotted ALOHA. These papers, however, have not considered the capture effect or MPR from one collision. Furthermore, a significantly large buffer is required at the receiver to hold all the received signals until the included packets are resolved.

As originally pointed out in [9], the power allocation enhances the resulting throughput of the system. In [11], [16], a random transmission power control was proposed in order to achieve higher throughput assuming that the receiver is able to decode a single packet from the collision. In [27], a decentralized power control for random access with SIC was proposed; in this system, the discrete power levels were derived, and the optimal probability distribution of power levels was obtained, with the implicit limitation of that, at most, two packets could be decoded from a single collision.

In this paper, we focus on a random power allocation to enhance the MPR capability and the use of SIC for the sin-
ingle random access time slot to avoid the large buffer at the receiver side. Specifically, we propose decentralized multi-level power allocation (MLPA). The users randomly select transmission powers from the given discrete power levels according to the corresponding probability distribution and the receiver tends to decode more than two packets from a collision. This decentralized power control and optimization are similar to [27] but our approach can be considered as a more general case. We propose a suboptimal per-level iterative optimization method for the additive white Gaussian noise (AWGN) channel. However, the obtained power levels and the corresponding probabilities also perform well in the fading environment. The main contributions of this paper are summarized as follows:

- The probability distribution optimization problem of the multilevel power allocation for generalized random access with MPR and SIC is formulated.
- A feasible suboptimal solution using the virtual user method and the per-level iterative optimization process is proposed.
- The theoretical throughput is derived from the optimization results and is used to investigate the improvement of throughput with an increase in the power level.
- The selection of base code is investigated by performing computer simulations.

The paper is organized as follows. Section 2 describes the system model. In Sect. 3, we propose the multilevel random access scheme, formulate the optimization problem, introduce the virtual user method, and present the iterative per-level optimization process. Section 4 is devoted to the analysis of the proposed scheme and provides the analytical value of the system MAC throughput. In Sect. 5, we compare the performances by presenting the numerical results from computer simulations. Finally, Sect. 6 summarizes our conclusions.

2. System Model

Preliminary to an explanation of the system model treated throughout the paper, the notation is summarized in Table 1.

We consider a wireless random access network that consists of one common base station (BS) and $K$ users as illustrated in Fig. 1. Let $K = \{1, 2, \cdots, K\}$ denote the set of users and $k \in K$ is an index of the user. As shown in Fig. 2, the information packets are first encoded by sufficiently powerful channel codes, such as turbo codes and low-density parity-check (LDPC) codes, and then the coded packets are modulated to complex signals. The channel coding and the modulation together constitute the base code with the data rate $R_D$.

At every time slot, each user randomly selects a transmission power and transmits its own coded packet with the chosen power. Namely, user $k$ transmits with power $e_k \in [0, 1]$. If $e_k = 0$, user $k$ is idle during the time slot, and if $e_k = 1$, user $k$ transmits with full power. Note that this can be considered to be a simplified case of the slotted ALOHA, where each user either transmits with full power or keeps idle. Each user selects the transmission power independently in a distributed manner, and none has any knowledge of the transmission power of the other users. The network is assumed to be fully loaded, that is, each user always has a packet to transmit and the buffer is assumed to be sufficient (i.e., the arrival packets are queued in the buffer that is large enough that packets are never discarded).

The received signal $y$ at the BS can be written as

$$y = \sum_{i=1}^{K} h_i \sqrt{e_i} x_i + z,$$

where each user either transmits with full power or keeps idle. Each user selects the transmission power independently in a distributed manner, and none has any knowledge of the transmission power of the other users. The network is assumed to be fully loaded, that is, each user always has a packet to transmit and the buffer is assumed to be sufficient (i.e., the arrival packets are queued in the buffer that is large enough that packets are never discarded).

The received signal $y$ at the BS can be written as

$$y = \sum_{i=1}^{K} h_i \sqrt{e_i} x_i + z,$$  (1)
where $x_i$ is the transmitted signal of the $i$-th user with amplitude $\sqrt{e_i}$ and $h_i$ denotes the complex channel coefficient. When additive white Gaussian noise (AWGN) channels is considered, we have $h_i = 1$ for all $i$. Also, when flat Rayleigh fading channel is considered, the channel coefficient is modelled as a circularly symmetric complex Gaussian random variable $h_i \sim \mathcal{CN}(0, 1)$. We assume that the channel state information (CSI) is ideally available at the BS but not at the users. The noise $z$ is modelled as a circularly symmetric complex Gaussian random variable $z \sim \mathcal{CN}(0, N_0)$. One of the application scenarios is that the users are located far away from the BS and they are close to each other compared with the distance to the BS. Hence the path loss difference of uplink (user to BS) is not the dominant factor for the decentralized power allocation and is neglected in this paper.

Then, the signal-to-interference-and-noise ratio (SINR) corresponding to the user $k$ can be calculated by

$$\text{SINR}_k = \frac{|h_k|^2 e_k}{\sum_{i \in K \setminus k} |h_i|^2 e_i + N_0},$$

where $K \setminus k$ denotes the subset of $K$ from which $k$ is excluded. The BS can decode the packet of user $k$ if and only if $\text{SINR}_k$ exceeds the decoding threshold $\rho$ which is obviously defined by $R_o$. The threshold of ideal channel coding $\tilde{\rho}$ is thus given by

$$\tilde{\rho} = 2^{R_o} - 1.$$ (3)

However, owing to the gap between practical codes and channel capacity, a higher SINR is required to correctly decode the information. Hence, we use an arbitrary small $\Delta > 0$ to consider this gap. The practical decoding threshold is given by

$$\rho = \left(2^{R_o} - 1\right) (1 + \Delta).$$ (4)

In a communication system, the packets from different users would correlate each other and significantly degrade the decoding performance. To deal with this problem, bit-interleaved coded modulation (BICM) that originally proposed to improve the decoding performance in the fading environment [28], can be used to eliminate the correlations by letting each packet use statistically independent interleaver. We perform simulations of transmitting totally correlated packets but without the noise, comparing with the case of AWGN channel that the packet is only corrupted by the noise. As shown in Fig. 3, the correlated interferences profoundly degrade the decoding performance. However, with the BICM, the cases that the users using different interleavers achieve close waterfall region to the case of the AWGN channel. In a practical scenario, the users would generate partially correlated packets but the correlation portion is small, and the BICM can further decorrelate the packets. Hence, we use the decoding threshold in the proposed algorithm and the simulations.

Upon decoding the information packet with the highest SINR, the SIC process begins to subtract the corresponding packet from the received signal. The SIC process is repeated in the order of descending SINR until all packets have been decoded except for that none’s SINR exceeds the decoding threshold.

3. Multilevel Power Allocation

3.1 Problem Statement

We consider the discrete power levels and the corresponding probability mass function (PMF) in a random access system, since it was conjectured in [16], [27] that the optimal power distribution may be of a discrete nature. For each time slot, each user randomly selects a transmission power from a set of discrete power levels $E = \{E_0, E_1, \cdots, E_L\}$, according to the discrete probability distribution $p = [p_0, p_1, \cdots, p_L]$. Here $l \in \{0, 1, \cdots, L\}$ is the index of the power levels. Taking user $k$ for example, the probability that it transmits with $E_l$ is given by

$$p_l = \Pr(e_k = E_l).$$ (5)

With the power constraints, the power of the lowest level is $E_1 > 0$ and the power of the highest level is $E_L \leq 1$. We assume that $E_i < E_j$, $\forall i < j$. When $e_k = E_0 = 0$, it means that the user does not transmit signals, and the corresponding probability is $1 - \sum_{l=1}^L p_l \geq 0$; hence $\sum_{l=1}^L p_l \leq 1$. The key issues here are how to design the discrete power levels and how to optimize the probability distribution in order to maximize the average number of decodable packets, given the constraint on the transmission power. In the rest of paper, this average number of decodable packets is referred to as MAC throughput.

3.2 Optimization Problem Formulation

To formulate the problem of maximizing the system MAC throughput, we define the event of successful decoding the
users with power level \( l \) as

\[
S_l = \text{Event}[\text{successfully decode } K_l \text{ packets of } l\text{-th level}],
\]

(6)

where \( K_l \) is the number of users transmitting with power \( E_i \) and \( \sum_{l=0}^{K} K_l = K \).

For analytical tractability, we here consider the AWGN channels, namely \( h_i = 1, \forall i \). Consider the decoding of an arbitrary user with the \( l \)-th power level in the sufficiently high SNR region where the effect of noise can be neglected.

Assuming that the users above the \( l \)-th power level have been successfully decoded and ideally subtracted by the SIC process, the probability of successful decoding is given by

\[
q_l(p, E) = \Pr(S_l|S_{l+1}, \ldots, S_L) = \Pr\left(\frac{E_i}{(K_i - 1)E_i + \sum_{l=1}^{K_i} E_i} \geq \rho\right),
\]

(7)

where the other \((K_i - 1)\) signals with the same power \( E_i \) and the signals with the lower power levels \( \sum_{l=1}^{K_i} E_i \) are considered to be interference. Equation (7) shows the condition for successful decoding one user from among \( K_i \) users. Upon the successful decoding of the first packet with power level \( E_i \), the interference from the same power level is reduced to \((K_i - 2)E_i\), due to the packet subtraction. Hence with this condition, all \( K_i \) users can be successfully decoded.

Assuming the packets of higher levels are successfully decoded and subtracted, for the \( l \)-th power level, the conditional expectation of packet number that can be successfully decoded is given by\(^\dagger\)

\[
d_l(p, E) = \mathbb{E}[K_lq_l] = \sum_{K_l=1}^{K} K_l\Pr(K_l)\Pr\left(\frac{E_i}{(K_i - 1)E_i + \sum_{l=1}^{K_l} E_i} \geq \rho\right),
\]

(8)

where \( K_l \) is a binomial random variable whose probability is given by

\[
\Pr(K_l) = \binom{K}{K_l} p_l^l (1 - p_l)^{K-K_l}.
\]

(9)

Let \( D_l(p, E) \) denote the expectation of packet number that can be successfully decoded from power level 1 to \( L \). Considering the overall SIC process, the decoding starts from the highest power level \( L \). The throughput contribution of the \( L \)-th power level is simply given by \( d_L \), since there is no higher power level. For power levels \( l < L \), we need to take into account of the probabilities of successfully decoding the higher levels since only when the packets of all the higher levels are subtracted first, the packets of \( l \)-th level have the chance to be decoded. Hence the contribution of the \( l \)-th power level is given by \( d_l \prod_{l=1}^{L} q_l \). Finally the overall throughput \( D_L(p, E) \) is a summation of the throughput contributions from all the power levels, as the following

\[
D_L(p, E) = d_L + \cdots + d_l \prod_{l=1}^{L} q_l, \quad l = 1, 2, \ldots, L
\]

\[
= d_L + \sum_{j=1}^{L-1} d_j \prod_{l=1}^{L} q_l.
\]

(10)

For a general case, let \( D_l(p, E) \)\(^\dagger\) denote the expectation of packet number that can be successfully decoded from power levels 1 to \( L \), assuming the packets of all the higher power levels are subtracted. Similarly \( D_l \) is given by

\[
D_l(p, E) = d_l + \sum_{j=1}^{L-1} d_j \prod_{l=1}^{L} q_l.
\]

(11)

For the multiple-access network, the system MAC throughput \( T \) is used to measure the MAC layer efficiency, which is mathematically defined by

\[
T = \frac{\sum_{i=1}^{N} D_L(i)}{N}.
\]

(12)

where \( N \) is the number of time slots and \( D_L(i) \) is the number of successfully decoded packets in the \( i \)-th time slot. According to the definition of \( T \), the optimization of the function \( D_L(p, E) \) is identical to the maximization of the system MAC throughput. Direct formulation of the optimization problem is hence given by

\[
\max_{p, E} D_L(p, E)
\]

\[
\text{s.t.} \quad 0 < E_i \leq 1, \quad i = 1, \ldots, L
\]

\[
E_{i-1} - E_i < 0, \quad i = 1, \ldots, L
\]

\[
0 < p_i < 1, \quad i = 1, \ldots, L
\]

\[
\sum_{i=1}^{L} p_i \leq 1.
\]

(13)

### 3.3 Suboptimal Per-Level Optimization

Since the number of power levels \( L \) is unknown, direct optimizing the target function \( D_L(p, E) \) is unfeasible. Moreover, even if we can derive \( L \), it is difficult to prove the concavity of the target function \( D_L(p, E) \), which consists of multiple functions of \( q_l(p, E) \) and \( d_l(p, E) \).

By observing (7), we notice that the summed interference \( \sum_{i=1}^{K} K_i E_i \) makes it difficult to derive the successful decoding probability \( q_i \). If this item can be replaced by a function of \( E_i \), it may be possible to obtain a closed form of \( q_i \) that depends only on \( E_i \) and \( K_i \). Hence to simplify the problem and make the optimization feasible, we propose a suboptimal method that introduces relationships among the power levels. Specifically, except for the first power level, the relationships among the current \( l \)-th power level \((l > 1)\) and its lower power levels is given by

\(^\dagger\)To simplify the notation, \( q_l(p, E) \) will be written as \( q_l \) when this will not cause any confusion.

\(^\dagger\)\( D_l(p, E) \) will be written as \( D_l \) when this will not cause any confusion.
Based on the law of large numbers, for large $K$, we can make the following approximation:

$$
\sum_{i=1}^{l-1} K_p E_i \approx \sum_{i=1}^{l-1} K E_i.
$$

(15)

From (14) and (15), the total power of all users at lower levels can be treated as a single virtual user at the current power level. Thus, (7) can be rewritten as

$$
q(p_i) = \text{Pr} \left( \frac{E_i}{(K_l - 1)E_l + \sum_{i=1}^{l-1} K E_i} \geq \rho \right)
$$

$$
\approx \text{Pr} \left( \frac{E_i}{(K_l - 1)E_l + E_i} \geq \rho \right)
$$

$$
= \text{Pr} \left( K_l \leq \frac{1}{\rho} = \text{Pr} (K_l \leq \nu) \right)
$$

$$
= \sum_{u=0}^{\nu} \binom{K}{u} p_i^u (1 - p_i)^{K-u} (u + D_{l-1}),
$$

(16)

where $\nu = \left\lfloor \frac{1}{\rho} \right\rfloor$. Since $K_l$ is the number of packets, and thus it is always an integer, the equation $\text{Pr} (K_l \leq 1/\rho) = \text{Pr} (K_l \leq \nu)$ holds. Hence for $l > 1$, at most $\nu$ users at the $l$-th power level can be decoded. With this approximation, for each power level, using (7)-(11) and (16), the target function $D_l(p,E)$ is reduced to a simplified function $D_l(p)$, which is given by

$$
D_l(p) = q_l(K_l + D_{l-1})
$$

$$
= \text{Pr} (K_l \leq \nu) (K_l + D_{l-1})
$$

$$
= \sum_{u=0}^{\nu} \binom{K}{u} p_i^u (1 - p_i)^{K-u} (u + D_{l-1}),
$$

(17)

where $p_i$ is the only variable and $D_{l-1}$ is set to a constant, since $D_{l-1}$ is maximized for power level $l - 1$ and is independent of $p_i$. As shown in (16), the set of power levels $E$ has no effect on the target function. Finally, for the $l$-th power level, the optimal $p_i^*$ can be obtained by maximizing the function $D_l(p_i)$, which turns out to be quasiconcave.

**Proposition 1.** Function $D_l(p_i)$ is quasiconcave.

The proof is given in Appendix.

Since $D_l(p_i)$ is proven to be quasiconcave, we know that there is one and only one global maximal for $D_l(p_i)$, and the optimal $p_i^*$ can be obtained by the optimization algorithm. Hence the optimization of the multilevel power allocation becomes feasible by using an iterative process from the lowest to the highest power level. For each power level, the optimization problem is simplified to

$$
\max_{p_i} D_l(p_i) = \sum_{u=0}^{\nu} \binom{K}{u} p_i^u (1 - p_i)^{K-u} (u + D_{l-1}),
$$

s.t. $0 < p_i < 1$.

(18)

### Algorithm 1 Obtain $E$ and $p^*$

**Initialization:** $l = 1, E_1$

**while** $l$ do

- Compute $D_l$ and obtain $p_i^*$ using (17)

  - **if** $\sum_i p_i^* < 1$ **then**
    - save $p_i^*$ to $p^*$, continue
  - **else**
    - discard $p_i^*$, break

- **end if**

- Calculate $E_l$ using (14)

  - **if** $E_l < 1$ **then**
    - save $E_l$ to $E$, $l = l + 1$, continue
  - **else**
    - discard $E_l$ and $p_i^*$, break

- **end if**

**end while**

The optimization process begins from the lowest power level $l = 1$, where the minimum required transmission power is $E_1 = N_0 \rho$. For higher power levels $l > 1$, the power is calculated by (14). The entire process is shown as Algorithm 1. The optimization is calculated iteratively up to the highest power level $L$, with constraints on the power $E_l \leq 1$ and on the probability $\sum_{i=1}^{L} p_i \leq 1$. Finally, we can obtain the set of power levels $E$ and the corresponding optimized discrete probability distribution $p^*$.

### 3.4 Calculation of Power Levels

In this subsection, we provide a method for choosing the parameters that improve the calculation of $E_l$ by taking into account the effect of noise. For a given SNR, the system MAC throughput depends only on the base rate, since for a given base rate $R_n$, the decoding threshold $\rho$ and $\nu$ are determined. However, we notice that $\nu \leq 1/\rho$, and the margin ratio of the decoding threshold is

$$
\delta = \frac{1}{\rho} - \nu,
$$

(19)

where $0 \leq \delta < 1$.

In the presence of noise, the condition in (16) for successful decoding can be rewritten as

$$
\frac{E_i}{(K_i - 1)E_i + E_i + N_0} \geq \rho,
$$

(20)

which can be expressed as

$$
K_i \leq \frac{1}{\rho} - \frac{N_0}{E_i}.
$$

(21)

Recalling (16), the condition for successful decoding after omitting the noise and excessive interference is $K_i \leq \nu$. The effect of noise can be neglected due to the margin ratio if

$$
\nu \leq \frac{1}{\rho} - \frac{N_0}{E_i}.
$$

(22)

For the power levels of $l > 1$, it follows that

$$
E_l \geq \frac{N_0}{\delta}.
$$

(23)
We use these requirements to improve the calculation of power for $l > 1$, as follows:

$$E_l = \max \left( \sum_{i=1}^{l} K p_i E_i, \frac{N_0}{\delta} \right).$$

(24)

4. Performance Analysis

In this section, we derive the analytical system MAC throughput from the probabilities $p^* = [p_0^*, p_1^*, \ldots, p_L^*]$ obtained by the proposed algorithm. The obtained power levels and the corresponding probabilities are shown in Fig. 4 and Fig. 5, respectively.

According to the calculation of $D_L$ in (10), we can derive the theoretical system MAC throughput $D_L^*$. Let $d_i^*$ denote the probability of successful decoding the $i$-th power level for $l \in \{1, 2, \ldots, L\}$. By replacing $p_l$ with $p_i^*$ in (7), we derive $q_i^*$ by the following

$$q_i^* = \Pr(S_i|S_{i+1}, \ldots, S_L, p^*)$$

$$= \sum_{u=0}^{K} \binom{K}{u} p_i^u (1 - p_i)^{K - u}$$

$$\approx \sum_{i=0}^{v} \frac{\lambda_i^u e^{-\lambda_i}}{u!}$$

where $\lambda_i = p_i^* K$, with $K$ sufficiently large, from the fact that a Poisson distribution is an approximated version of the binomial distribution for large $K$, we make the above approximation.

Let $d_i^*$ denote the MAC throughput contributed by power level $l$. Similarly, by replacing $p_l$ with $p_i^*$ in (8), we derive $d_i^*$ by the following

$$d_i^* = \sum_{u=1}^{v} \binom{K}{u} p_i^u (1 - p_i)^{K - u}$$

$$\approx \sum_{i=1}^{v} \frac{\lambda_i^u e^{-\lambda_i}}{(u - 1)!}$$

where $\lambda_i = p_i^* K$.

Using (10), (25) and (26), the analytical system MAC throughput is given by the following closed-form expression

$$D_L^*(d^*, q^*, L)$$

$$= d_L^* + \cdots + d_1^* \prod_{i=1}^{L} q_i^* + \cdots + d_1^* \prod_{i=2}^{L} q_i^*$$

$$= d_L^* + \sum_{j=1}^{L-1} \left( d_j^* \prod_{i=j+1}^{L} q_i^* \right)$$

(27)

where $q^* = [q_1^*, \ldots, q_L^*]$ and $d^* = [d_1^*, \ldots, d_L^*]$.

As shown in Fig. 6, the throughput results from the simulation are tightly bounded by the derived analytical throughput. We design the suboptimal algorithm with the assumption of low noise power and neglect the noise in the optimization problem formulation. Hence as the noise power decreases, the numerical throughput by the simulations approaches to the analytical throughput. Having the analytical throughput, we can well understand the performance that the random access system can achieve, especially for the case of low noise power.
Table 2  Simulation parameters.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Base channel code</td>
<td>1/5 turbo code</td>
</tr>
<tr>
<td>Base modulation</td>
<td>QPSK</td>
</tr>
<tr>
<td>Base rate $R_o$</td>
<td>$1/5 \times 2 = 0.4$</td>
</tr>
<tr>
<td>Gap of threshold $\Delta$</td>
<td>0.4125</td>
</tr>
<tr>
<td>Decoding threshold $\rho$</td>
<td>0.4467</td>
</tr>
<tr>
<td>Noise power $N_0$</td>
<td>$10^{-1}$</td>
</tr>
<tr>
<td>Fading channel</td>
<td>Flat Rayleigh fading</td>
</tr>
<tr>
<td>Number of user $K$</td>
<td>[4, 20]</td>
</tr>
</tbody>
</table>

5. Numerical Results

We perform computer simulations and use them to verify the effectiveness of the proposed MLPA scheme and to compare it with the existing random access schemes. The simulation parameters are listed in Table 2. We also perform simulations with using various data rates and provide some guidance on selecting the base code.

5.1 Throughput Performance Comparisons

We make the comparisons of the system MAC throughput using the the proposed scheme and the conventional decentralized power allocation schemes for various number of user $K$. Figure 7 shows the system MAC throughput performances of various random access schemes including the slotted ALOHA, the SIC scheme with decentralized power allocation (SIC-DPA) in [27], and our proposed SIC scheme with MLPA (SIC-MLPA). Specially, we also perform the simulation for the proposed MLPA with the same constraint as in [27] where at most two packets can be decoded from one collision (SIC-MLPA constrained).

The SIC-DPA scheme achieves superior throughput performance comparing with the slotted ALOHA, since the capability of SIC is exploited by properly allocating transmission power. However, its performance is limited, since at most two collided packets can be decoded, even if the decoding threshold is exceeded. The proposed SIC-MLPA achieves large throughput improvement without the constraint, and the SIC-MLPA constrained scheme still can achieve superior performance but with smaller gain. This indicates that the main gain comes from successful decoding from collisions of more than two packets. Hence to exploit the advantage of the SIC receiver, our proposal is more effective.

5.2 Throughput Performances in Fading Environment

Figure 8 shows the throughput performances of the random access schemes with SIC in the fading environment, using different decentralized power allocation strategies including the proposed SIC-MLPA scheme, the SIC-DPA scheme in [27], and the random access scheme with SIC but without transmission power allocation (the received power is changed randomly by the fading channel). The existing SIC-DPA scheme achieves superior throughput performance comparing with the scheme without power allocation, and the proposed SIC-MLPA further improves the throughput performance. These results show that although the fading affects the power allocation on the received powers, the power allocation schemes can still outperforms the scheme without power allocation.

Figure 9 compares the throughput performances between the fading channel and the AWGN channel. Both the existing SIC-DPA scheme and the proposed SIC-MLPA scheme in the fading environment achieve superior throughput performance. For our proposal, this is because that the random fading coefficient makes the received powers of the same transmit power level more dynamic and creates additional opportunities of successful decoding.

5.3 Base Code Selection

Since the parameters including $\rho$, $\nu$, and $\delta$ that affect the performance are determined only by the base rate $R_o$, we performed simulations for various base rates. The system PHY throughput results for $1/N_0 = 20$[dB] are listed in Table 3. Note that we set $\Delta = 0.1$ for the assumption of even powerful channel code. Here, the system PHY throughput $R$ is used to measure the overall efficiency of both the MAC and PHY layers. Since all of the users adopt the same base code...
with data rate $R_o$, the system PHY throughput is $R = R_oT$. According to Table 3, for a low-rate base code, the decoding threshold $\rho$ is low, and thus more packets of the same power level (larger $\nu$) can be decoded. The margin ratio $\delta$ is also important for the system PHY performance, since it makes the system more tolerant of noise and the random interference from lower power levels. Without designing the parameters as in (24), the base code of $R_o = 1/7 \times 2$ achieved an inferior system PHY throughput performance ($R^{(1)}$ column) due to the small margin ratio $\delta = 0.15$. The effect of a small $\delta$ can be alleviated by the parameter design using (24), as shown in the $R^{(2)}$ column of Table 3. Hence when selecting the base code, we need to avoid one that makes $\delta$ small. We can find the base code that maximizes the system PHY throughput by using the results of the simulations: the base code of $1/8 \times 2$.

### 6. Conclusions

In this paper, we have proposed multilevel power allocation for a decentralized random access scheme with the capabilities of MPR and SIC; it is suitable for the M2M communication application in 5G. We formulated the problem of optimizing the discrete power levels and the probability distribution, and by introducing relationships among the different power levels, we obtained a feasible suboptimal iterative per-level optimization process. We derived the theoretical system MAC throughput from the optimized transmission probabilities. The numerical results confirmed that the system MAC throughput for our system was better than that of conventional schemes. As an area of future work, we will continue to improve the performance of the random access scheme by combining the power allocation for a single time slot and the SIC for multiple time slots.

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### References


**Appendix:** Proof of the Quasiconcavity of $D_l(v)$

For large $K \gg 1$ and small $p_t \ll 1$, the function $D_l(v)$ can be approximated by

$$D_l(v) = \sum_{i=0}^{v-1} \binom{K}{i} p_t^i (1 - p_t)^{K-i} (i + d)$$

$$\approx \sum_{i=0}^{v-1} \frac{i + d}{i!} \lambda e^{-\lambda},$$

where $d$ is a positive real number and $\lambda = K p_t$. The first-order and second-order derivatives of $D_l(v)$ are given by

$$D_l' = \frac{\Gamma(v, \lambda) - e^{-\lambda} \Gamma(v-1, \lambda + d + v)}{\Gamma(v)},$$

$$D_l'' = \frac{e^{-\lambda} \Gamma(v-2, \lambda (1 + d + v) - (d + v)(v - 1))}{\Gamma(v)}.$$  

We can derive the unique root for $D_l' = 0$:

$$\lambda^* = \frac{(d + v)(v - 1)}{1 + d + v}. \quad (A \cdot 4)$$

Since $(d + v)(v - 1)$ and $(-1 + d + v)$ are both positive, we have $D_l^* < 0, \lambda \in [0, \lambda^*)$ and $D_l^* > 0, \lambda \in (\lambda^*, \infty)$. Hence $D_l'$ is monotonically decreasing for $\lambda \in (0, \lambda^*)$ and reaches the minimal negative value at $\lambda = \lambda^*$. It is also obvious that $D_l'(0) > 0$. It can be proved that there is one and only one $\lambda^*$ that makes $D_l'(\lambda = \lambda^*) = 0$ and $D_l'(-\lambda < \lambda^*) > 0, D_l'(-\lambda > \lambda^*) < 0$. This proves the quasi-concavity of $D_l$. 

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